Hierarchical Performance Analysis of Uncertain Large Scale Systems

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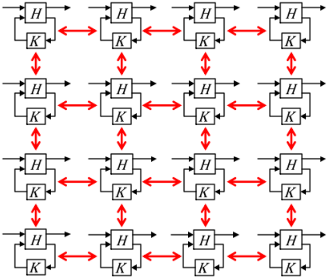
Context : PLL network

Large Scale Systems (LSS) : Phase Locked Loop (PLL) network

- PLL network to deliver clock signal to synchronous multi-core processors
- How to guarantee synchronization ?



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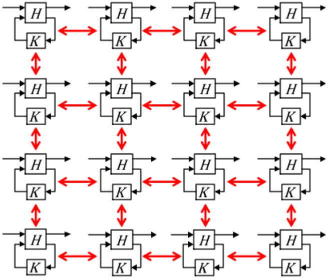

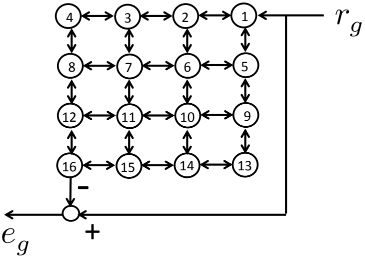

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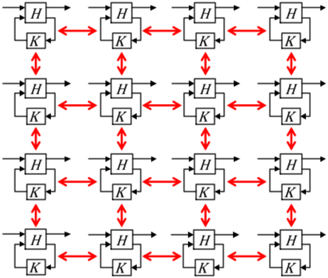
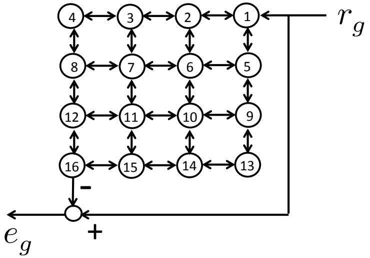
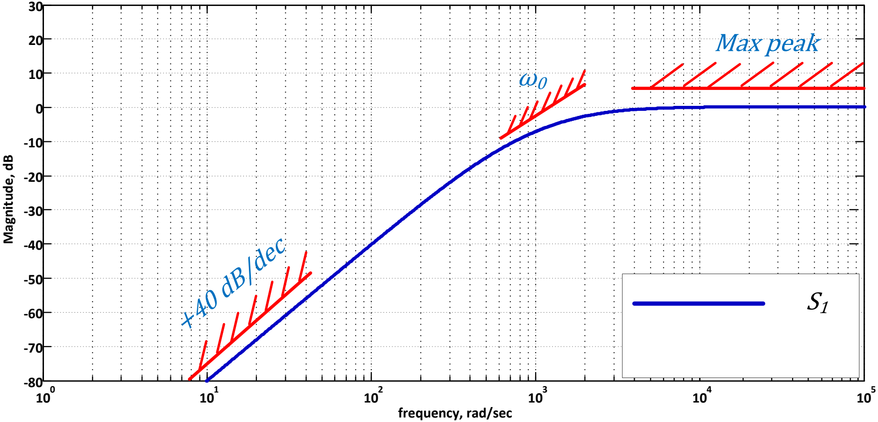




- Introduce global synchronization error
- Synchronization specifications (performance) are guaranteed if $T_{r_g \longrightarrow e_g}$ satisfies some frequency constraints

Context : Performance

Performance is expressed in frequency domain.



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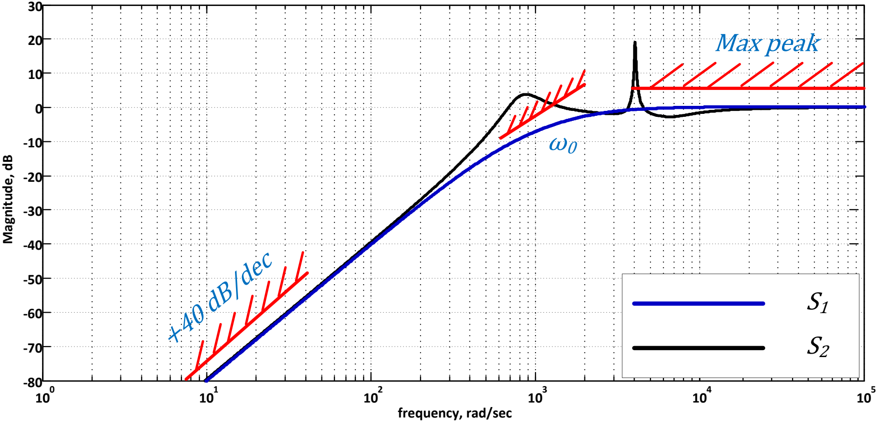

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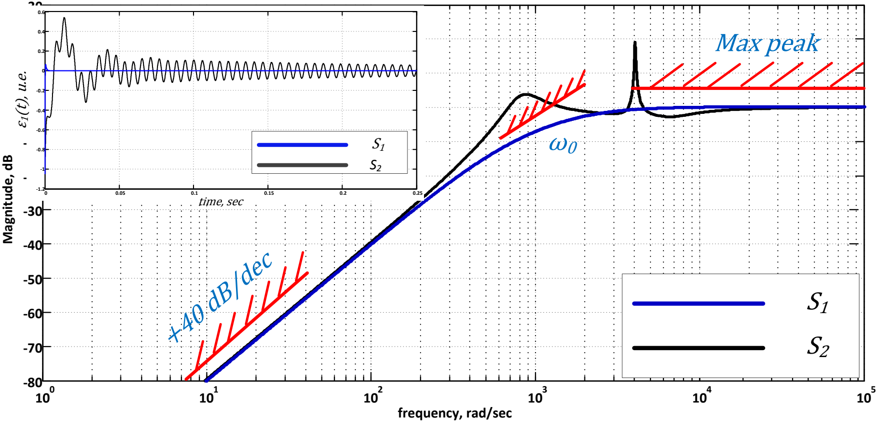

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Active clock distribution network

- Technological dispersions, modeling errors $\Longrightarrow$ uncertainties (Δ)

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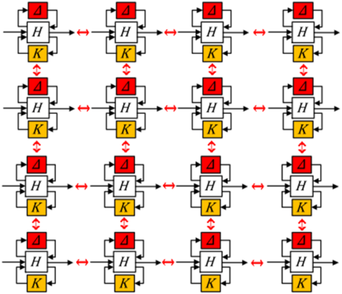

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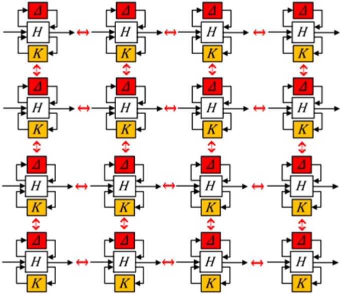

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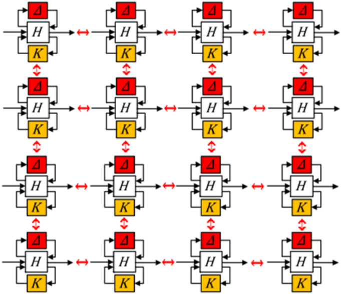
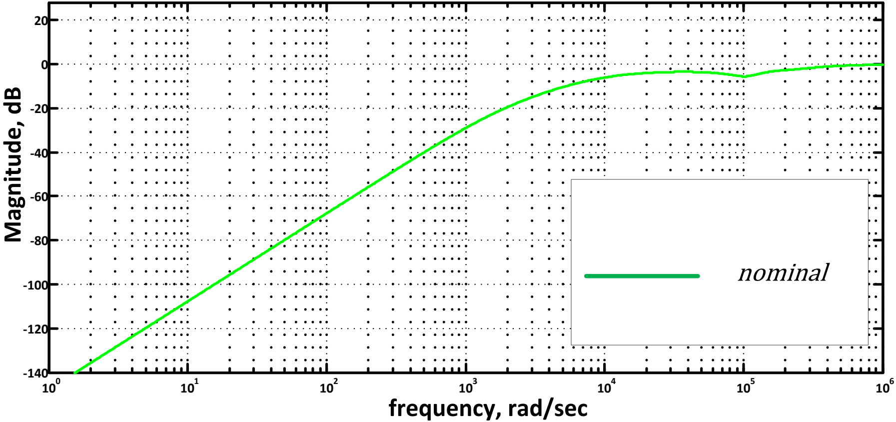


- Uncertain Network
- Robustness analysis :
  Perform the worst case robustness analysis for all the uncertainties Δ_i

Context : Performance



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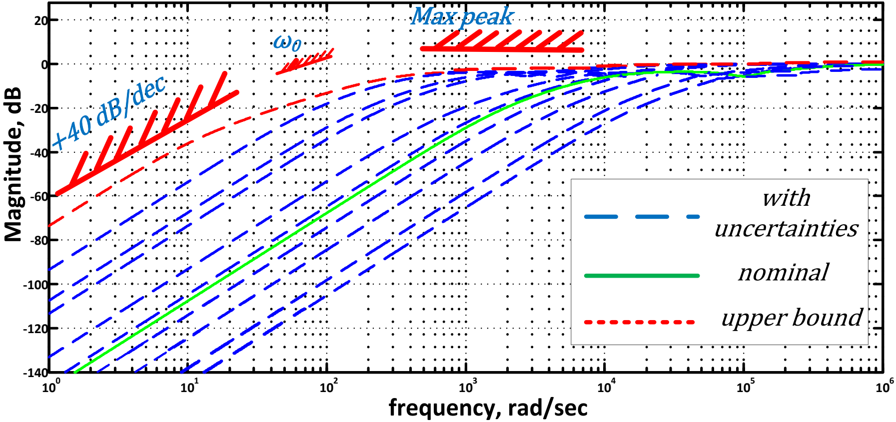
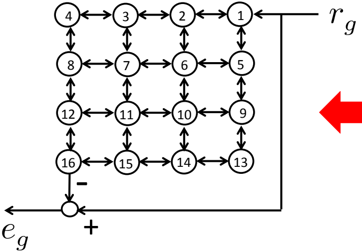
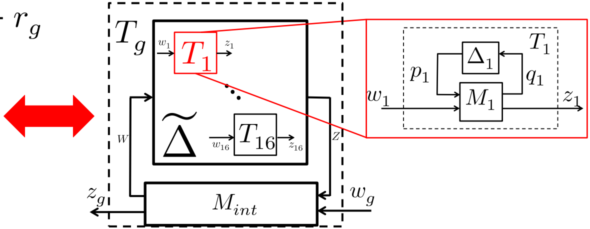


Synchronization specifications (performance) are guaranteed if the upper bound satisfies the frequency constraints

PLL network Performance

- 16 PLLs mutually synchronized





- Two uncertain parameters for every PLL $\Longrightarrow$ 32 uncertain parameters
- Nowadays networks : 100 PLLs $\Longrightarrow$ 200 uncertain parameters
  $\Longrightarrow$ classic method is not applicable
- 16 PLL network to show classic method results

Objective Compute an upper bound on $\|T_{r_g \to e_g}\|$ for all the uncertainties

Problem analysis

Large scale robustness analysis : two aspects problem

1. Robustness analysis : IQC based analysis (input-output description)
2. Large scale : decomposition techniques from graph theory

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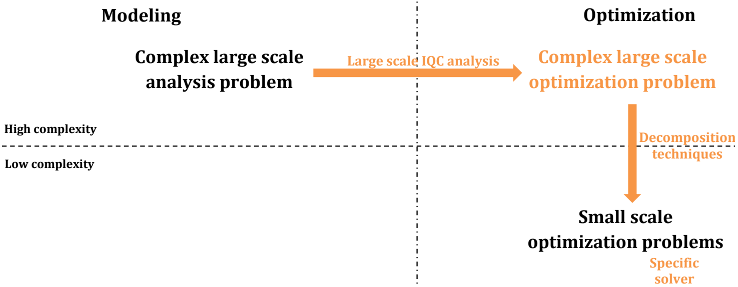
Few methods combining the two aspects : [Andersen et al., 2014]



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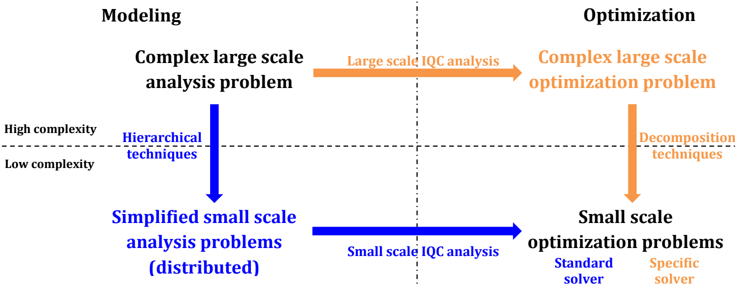

Integral Quadratic Constraints (IQC)

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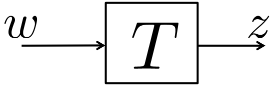
$$\int_{-\infty}^{+\infty} \begin{pmatrix} z(j\omega) \\ w(j\omega) \end{pmatrix}^* \Phi_P(j\omega) \begin{pmatrix} z(j\omega) \\ w(j\omega) \end{pmatrix} d\omega \geq 0$$



- Possibility to cover classical characterizations of performance
  - $\mathcal{L}_2$ gain

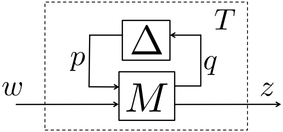
$$\int_0^{+\infty} \|z(t)\|_2 \; dt \leq \gamma^2 \int_0^{+\infty} \|w(t)\|_2 \; dt \iff \int_{-\infty}^{+\infty} \begin{pmatrix} z(j\omega) \\ w(j\omega) \end{pmatrix}^* \begin{pmatrix} -I & 0 \\ 0 & \gamma^2 \end{pmatrix} \begin{pmatrix} z(j\omega) \\ w(j\omega) \end{pmatrix} d\omega \geq 0$$

  - Passivity

$$\int_0^{+\infty} z(t)^T w(t) dt \geq 0 \iff \int_{-\infty}^{+\infty} \begin{pmatrix} z(j\omega) \\ w(j\omega) \end{pmatrix}^* \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \begin{pmatrix} z(j\omega) \\ w(j\omega) \end{pmatrix} d\omega \geq 0$$

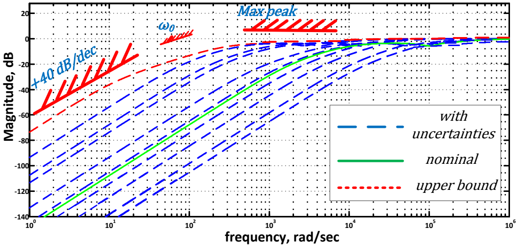
Proposed approach

Linear Time Invariant Systems



- $z(j\omega) = T(j\omega)w(j\omega)$ and QC based analysis
- Frequency domain : frequency response at ω_0
- Performance : compute an upper bound on the frequency response $(\bar{\sigma}(T) \leq \gamma)$

$$\min_{\gamma} \quad \gamma \quad \text{s.t.} \quad \begin{pmatrix} T \\ I \end{pmatrix}^* \begin{pmatrix} -I & 0 \\ 0 & \gamma^2 I \end{pmatrix} \begin{pmatrix} T \\ I \end{pmatrix} \geq 0$$



- General performance $\begin{pmatrix} T \\ I \end{pmatrix}^* \begin{pmatrix} X & Y \\ Y^* & Z \end{pmatrix} \begin{pmatrix} T \\ I \end{pmatrix} \geq 0$

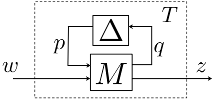
Proposed approach : Robust Performance Theorem (LTI systems)

QC for performance and uncertainty : Classical interpretation

Theorem (Robust Performance Theorem)

T is $\{X, Y, Z\}$ dissipative $i.e.$

$$\begin{pmatrix} T \\ I \end{pmatrix}^* \begin{pmatrix} X & Y \\ Y^* & Z \end{pmatrix} \begin{pmatrix} T \\ I \end{pmatrix} \geq 0 \quad \forall\, \Delta \in \underline{\Delta} \quad \Longrightarrow \textit{QC of } T$$



if and only if

1) $$\begin{pmatrix} \Delta \\ I \end{pmatrix}^* \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{12}^* & \Phi_{22} \end{pmatrix} \begin{pmatrix} \Delta \\ I \end{pmatrix} \geq 0 \quad \forall \Delta \in \underline{\Delta} \quad \Longrightarrow \textit{QC of } \Delta$$

2) $$\begin{pmatrix} M \\ I \end{pmatrix}^* \begin{pmatrix} -\Phi_{22} & 0 & -\Phi_{12}^* & 0 \\ 0 & X & 0 & Y \\ -\Phi_{12} & 0 & -\Phi_{11} & 0 \\ 0 & Y^* & 0 & Z \end{pmatrix} \begin{pmatrix} M \\ I \end{pmatrix} > 0$$

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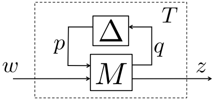

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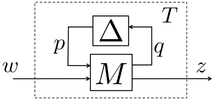

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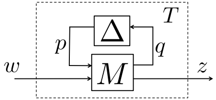

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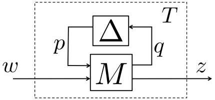

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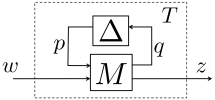

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- Conservatism depends on Φ_Δ

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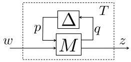

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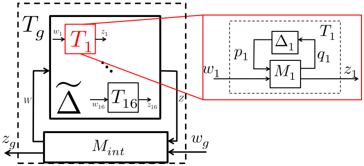

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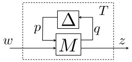

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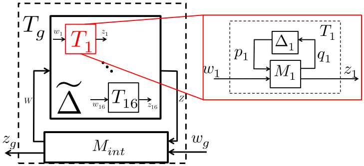

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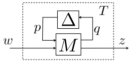

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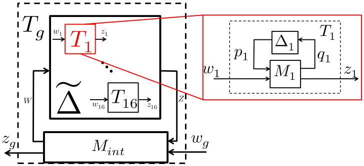

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QC for performance and uncertainty : New interpretation

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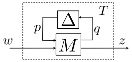

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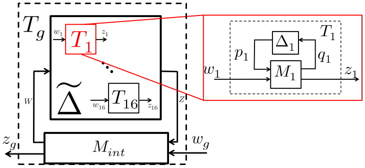

- Local step : find simple QC for every $T_i \Longrightarrow$ reduce the complexity
- T_i are seen as uncertainty Δ_i

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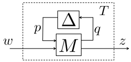

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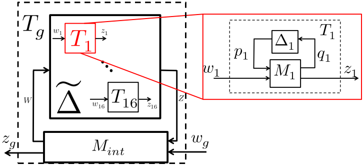
2) $$\begin{pmatrix} M \\ I \end{pmatrix}^* \begin{pmatrix} -\Phi_{22} & 0 & -\Phi_{12}^* & 0 \\ 0 & X & 0 & Y \\ -\Phi_{12} & 0 & -\Phi_{11} & 0 \\ 0 & Y^* & 0 & Z \end{pmatrix} \begin{pmatrix} M \\ I \end{pmatrix} > 0$$



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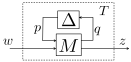
Proposed approach : Robust Performance Theorem (LTI systems)

QC for performance and uncertainty : New interpretation

Theorem (Robust Performance Theorem)

T is $\{X, Y, Z\}$ *dissipative* $i.e.$

$$\begin{pmatrix} T \\ I \end{pmatrix}^* \begin{pmatrix} X & Y \\ Y^* & Z \end{pmatrix} \begin{pmatrix} T \\ I \end{pmatrix} \geq 0 \quad \forall\, \Delta \in \underline{\Delta}$$



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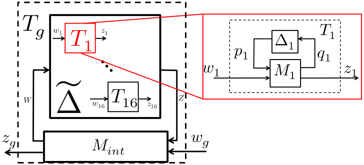

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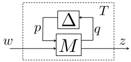

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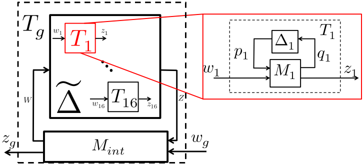

- Local step : find simple QC for every $T_i \Longrightarrow$ reduce the complexity
- T_i are seen as uncertainty Δ_i
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$\Longrightarrow$ create a basis for QC of T_i (to use as Φ_Δ in global step)

Proposed approach : Robust Performance Theorem (LTI systems)

Classical interpretation :

For given X, Y and Z find Φ from basis Φ_Δ

New interpretation :

- Find basis for X, Y and Z from given $\Phi \in \Phi_\Delta$
- Propagate the old basis into the new basis

$\Longrightarrow$ QC propagation

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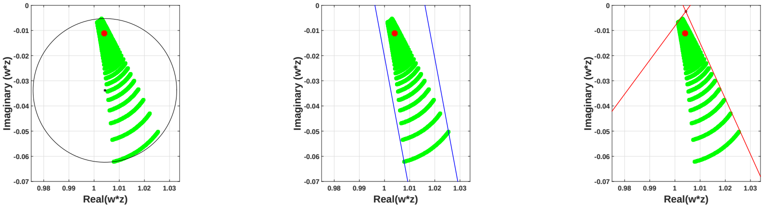
$\Longrightarrow$ QC propagation

Difficulties

- Size : not too big/small
- Quality : describes the best the uncertain system
- Efficient computation : convex

Robustness Analysis : QC classes

- Some classes of QC with geometric interpretations
  - disc [Dinh et al., 2013]
  - band [Dinh et al., 2014]
  - cone [Laib et al., 2015]



For given frequency ω_0, Complex plane (Real and Imaginary) : ● Nominal response and ● Uncertain response

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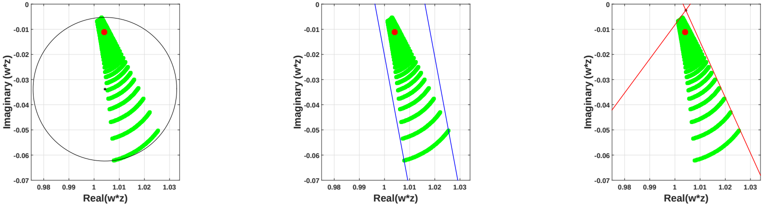

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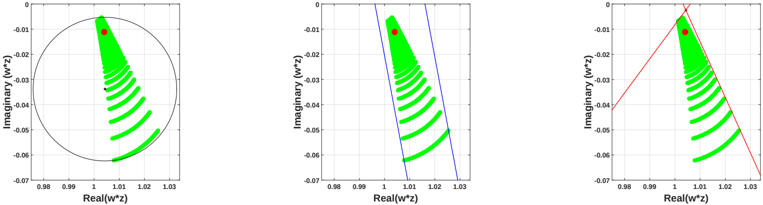

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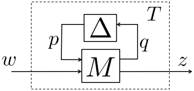
Robustness Analysis : Numerical Range

- For a given a frequency response Γ, at ω_0, of a system T



- The numerical range $\mathcal{N}(\Gamma)$

$$\mathcal{N}(\Gamma) = \{w^* z \mid z = \Gamma w, w \in \mathbb{C}^{n_w} \text{ and } \|w\| = 1\}$$

Robustness Analysis : Numerical Range

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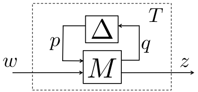

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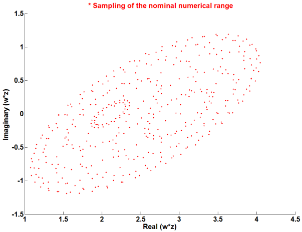

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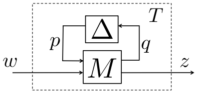

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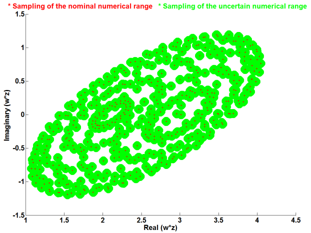
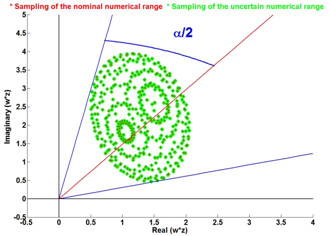
Certain numerical range   Uncertain numerical range



Robustness Analysis : Cone QC [Laib et al., 2015]

Theorem

Given the frequency response (at ω_0) of an uncertain system T



Finding the smallest α :

Robustness Analysis : Cone QC [Laib et al., 2015]

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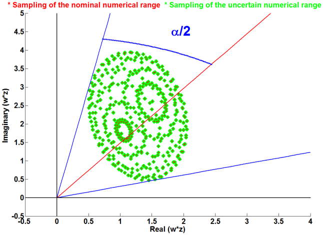

Finding the smallest α :

- *Quasiconvex optimisation problem*
- *LMI constraints*

Robustness Analysis : Cone QC [Laib et al., 2015]

Theorem

- *Given* $T = \Delta \star M$, *let* $\lambda = \cot \frac{\alpha}{2}$

$$\min_{\substack{\lambda,\,\Omega \\ \widehat{D}_1,\,\widehat{G}_1,\,\widetilde{D}_1,\,\widetilde{G}_1 \\ \widehat{D}_2,\,\widehat{G}_2,\,\widetilde{D}_2,\,\widetilde{G}_2}} \lambda$$

s.t :

$$\lambda \begin{pmatrix} \widehat{D}_1 & 0 \\ 0 & \widehat{D}_2 \end{pmatrix} + \begin{pmatrix} \widetilde{D}_1 & 0 \\ 0 & -\widetilde{D}_2 \end{pmatrix} > 0$$

$$\lambda \begin{pmatrix} M & 0 \\ I & 0 \\ 0 & M \\ 0 & I \end{pmatrix}^* \begin{pmatrix} B_1 & 0 \\ 0 & B_2 \end{pmatrix} \begin{pmatrix} M & 0 \\ I & 0 \\ 0 & M \\ 0 & I \end{pmatrix} + \begin{pmatrix} M & 0 \\ I & 0 \\ 0 & M \\ 0 & I \end{pmatrix}^* \begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix} \begin{pmatrix} M & 0 \\ I & 0 \\ 0 & M \\ 0 & I \end{pmatrix} > 0$$

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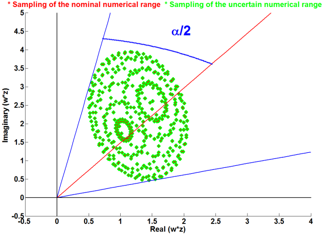
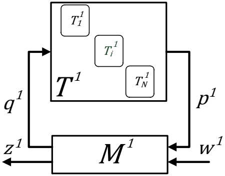


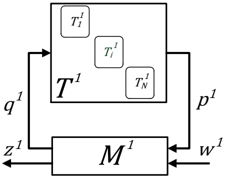
$\Longrightarrow$ Efficient tools to solve the problem

Summary



Level 1

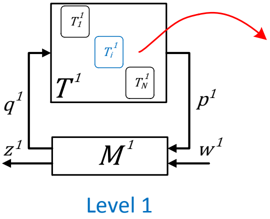
Summary



Level 1

1. Consider hierarchical structure of the system

Summary



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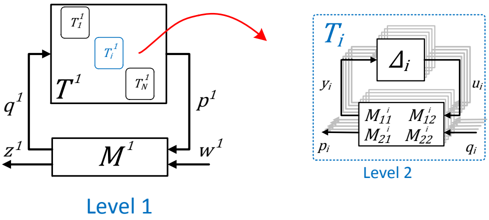

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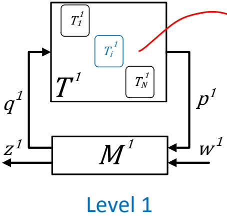
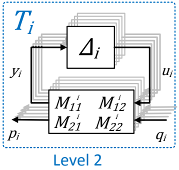
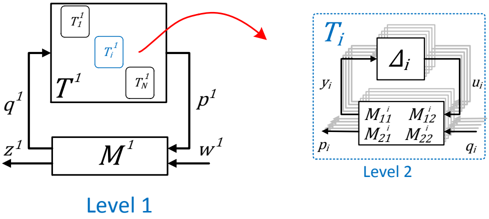




1. Consider hierarchical structure of the system
   - Find basis (QC description) for T_i with Robust Performance Theorem

Summary



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   - Find basis (QC description) for T_i with Robust Performance Theorem
   - Propagate this basis to the global level

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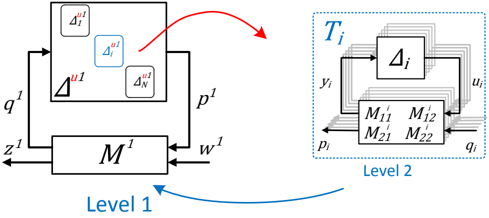
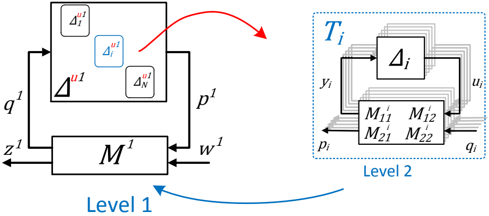


1. Consider hierarchical structure of the system
   - Find basis (QC description) for T_i with Robust Performance Theorem
   - Propagate this basis to the global level
2. For global hierarchical level, investigate the performance with Robust Performance Theorem

Summary



Computation time is reduced however conservatism may appear

- robustness of feedbacks loops $\Longrightarrow$ simple set may be sufficient
- combination of several simple sets $\Longrightarrow$ decrease of the conservatism
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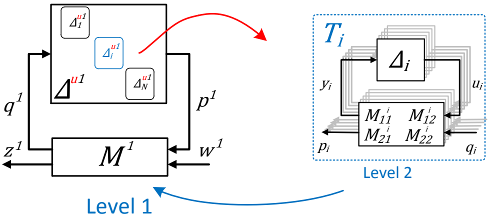

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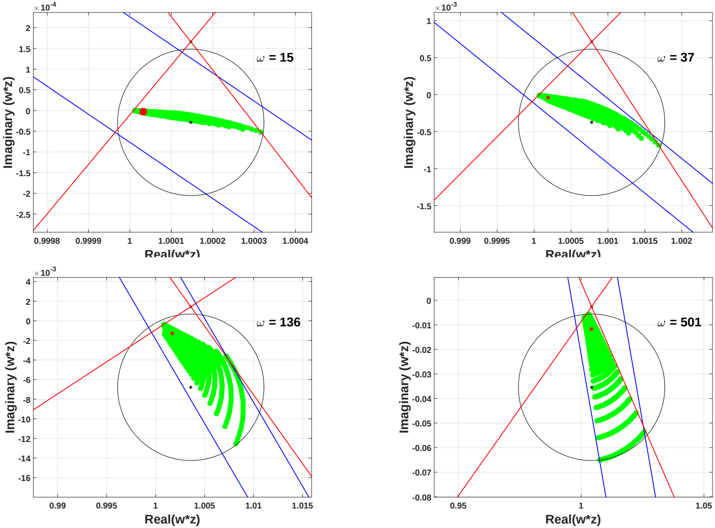
$\Longrightarrow$ trade-off conservatism/computation time

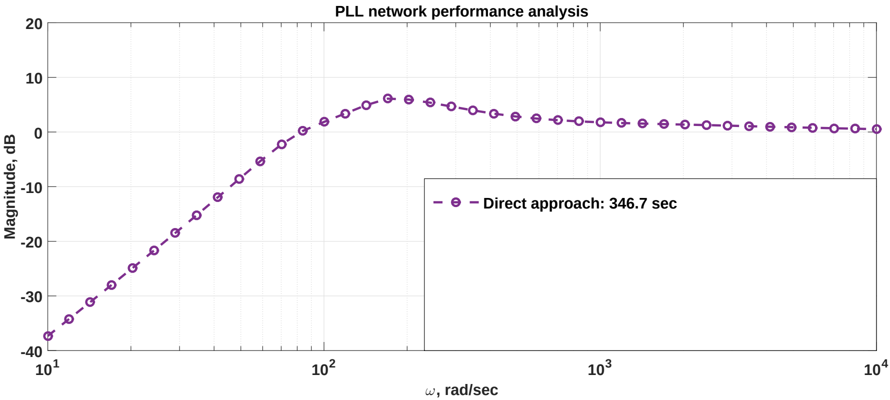
PLL network : Local Step

Characterize each PLL with QC with : disc, band and cone



PLL network : Global Step

Compute an upper bound on $T_{r_g \to e_g}$ for all the uncertainties



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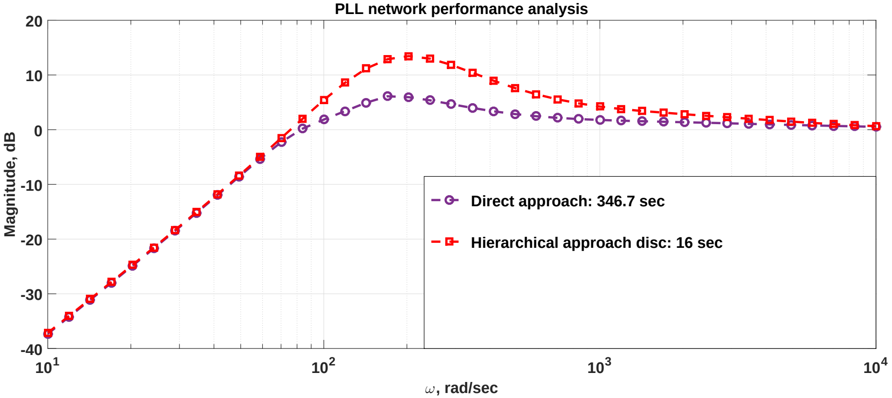

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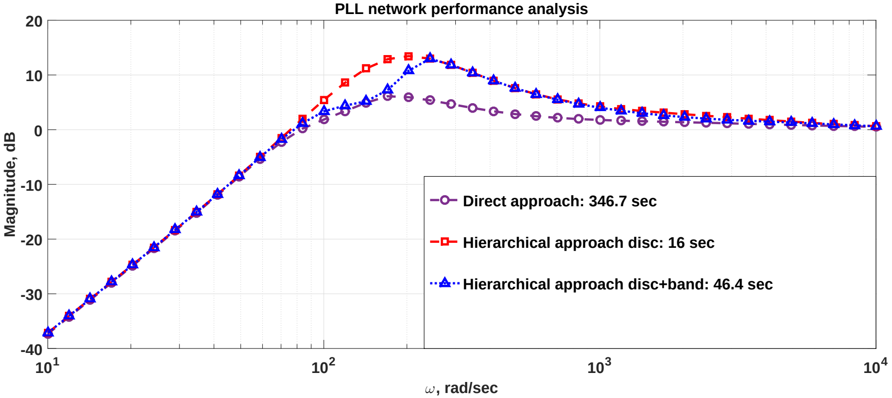

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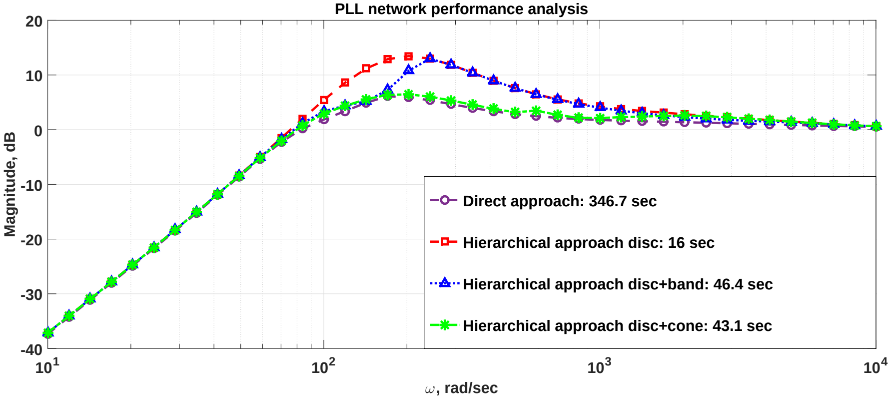

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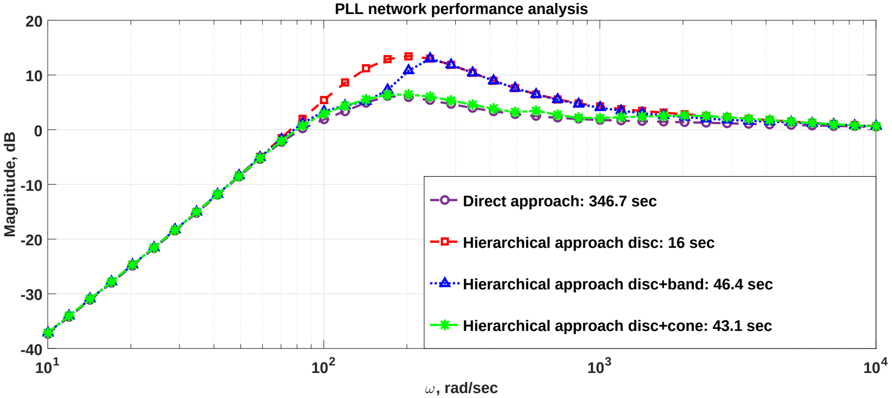
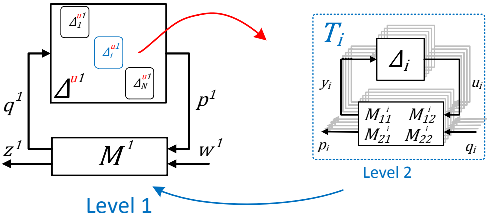


$\Longrightarrow$ Good choice of the basis elements

General Hierarchical Approach

Hierarchical approach



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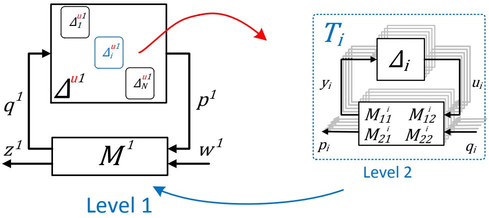
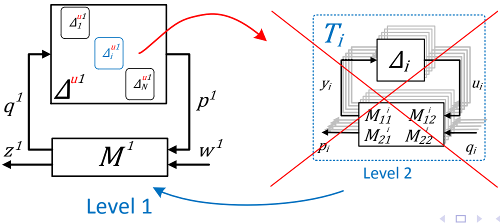


Special case : Direct approach



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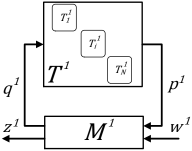
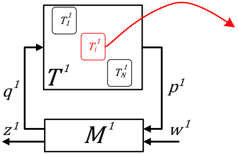


Level 1

General Hierarchical Approach



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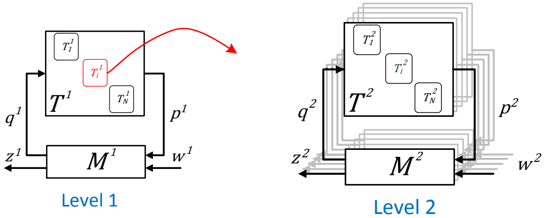

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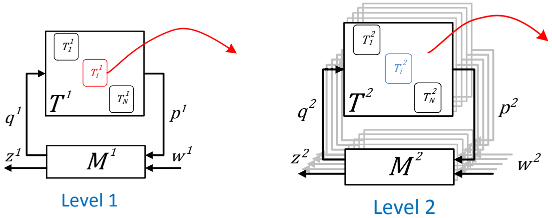

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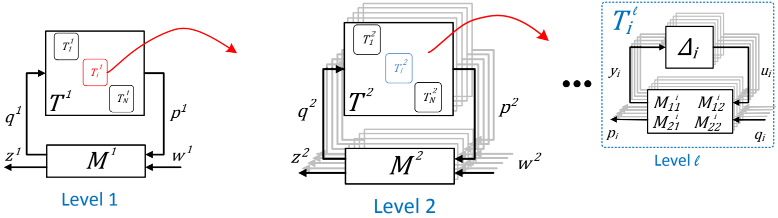

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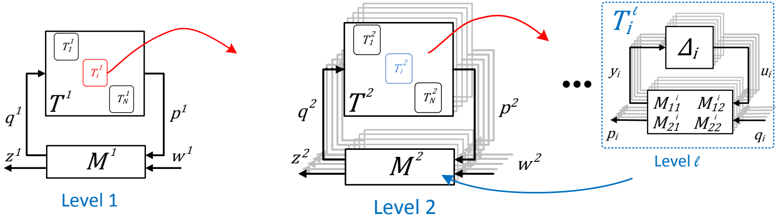

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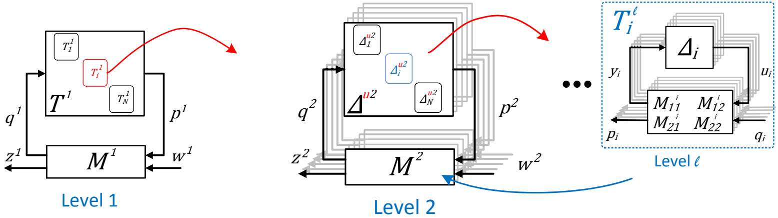

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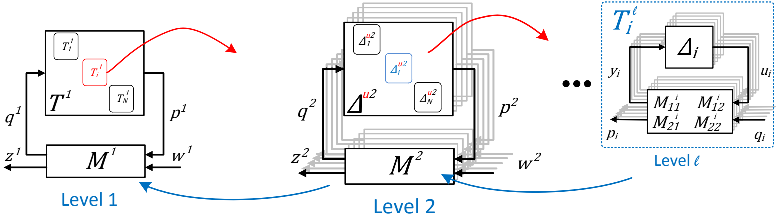

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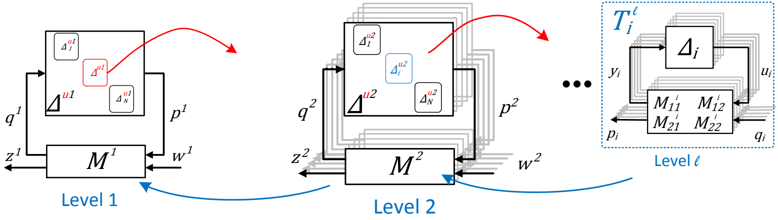

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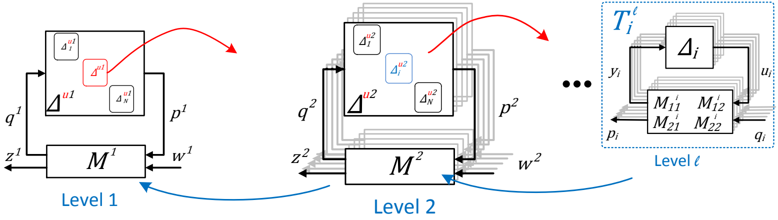
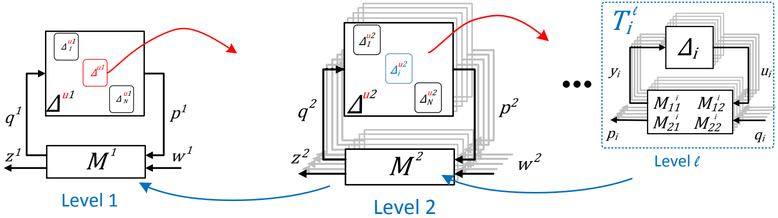


Many degrees of freedom to handle the trade-off conservatism/computation time

General Hierarchical Approach

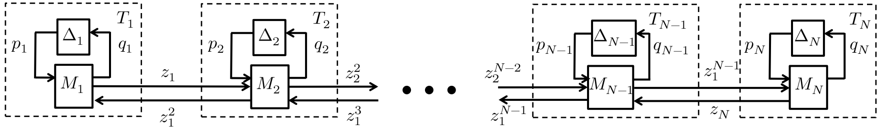


Many degrees of freedom to handle the trade-off conservatism/computation time

- Number of levels
- Number of T_i in each level
- Basis for Δ_i
- Basis for T_i in each level
- Parallel computing

Robust stability

Network with N systems randomly generated [Andersen et al., 2014].



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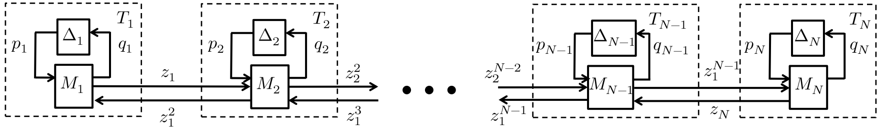

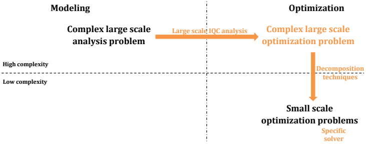

Robust stability

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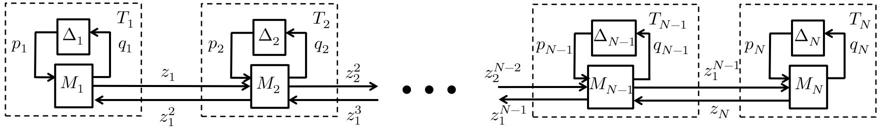
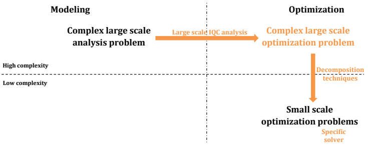
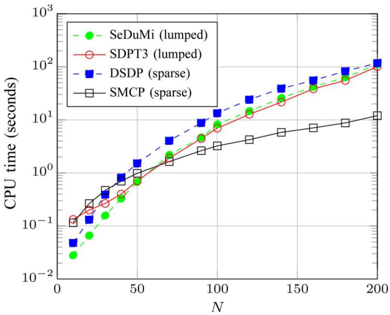
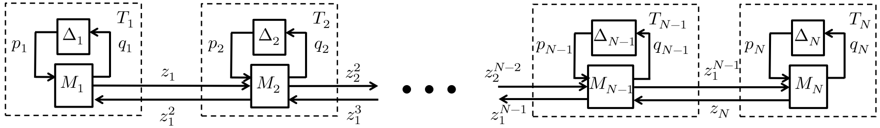




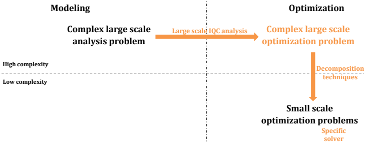


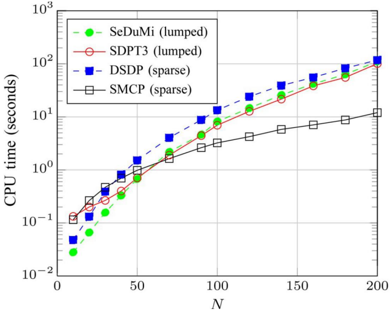
Robust stability

Network with N systems randomly generated [Andersen et al., 2014].



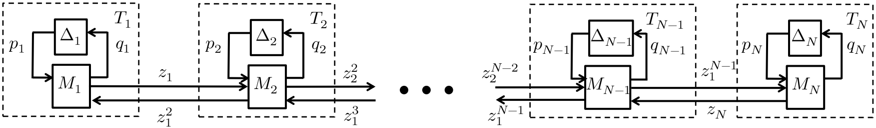
$$\frac{\text{Direct method computation time}}{\text{Proposed method computation time}} = 10 \text{ for } N = 200$$

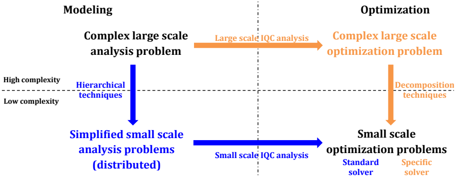




Robust stability

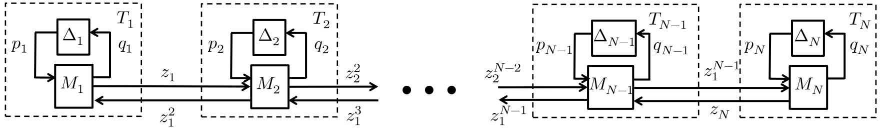
Network with N systems randomly generated [Andersen et al., 2014].

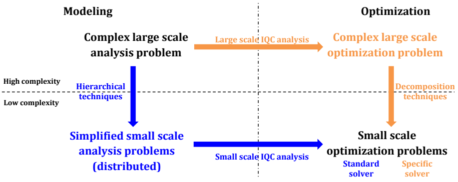


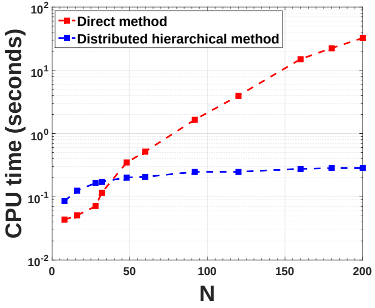


Robust stability

Network with N systems randomly generated [Andersen et al., 2014].

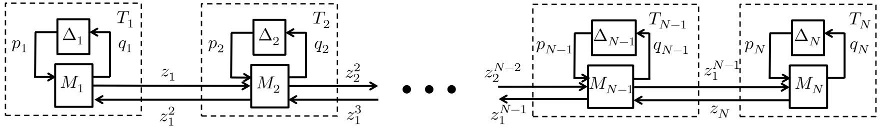




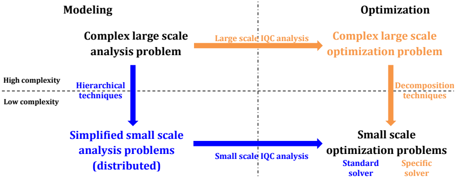


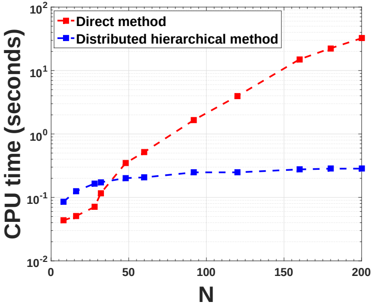
Robust stability

Network with N systems randomly generated [Andersen et al., 2014].



$$\frac{\text{Direct method computation time}}{\text{Proposed method computation time}} = 113 \text{ for } N = 200$$





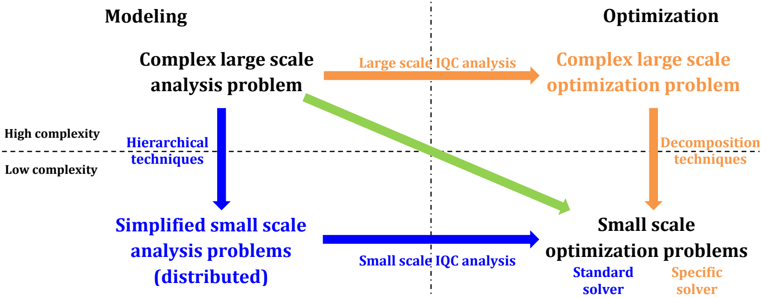
Conclusion

- Performance analysis of uncertain large scale systems
- Important computation time with direct method
- Exploit hierarchical structure using basis (QC) propagation
- General approach with degrees of freedom
- Reduce computation time with possible conservatism
- Trade-off conservatism/computation time

Perspectives

Perspectives

- Systematic decomposition technique using Graph Theory
- Combine hierarchical method with specific solvers



Thank you for your attention

Any Questions ?

Andersen, M., Pakazad, S., Hanson, A., and Rantzer, A. (2014).
Robust stability analysis of sparsely interconnected uncertain systems.
IEEE Transactions on Automatic Control, 59(8) :2151–2156.

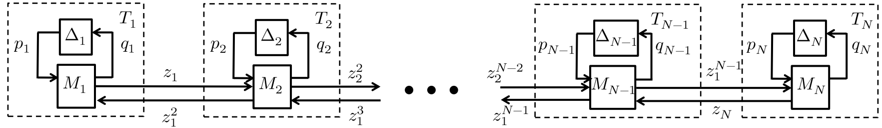
Dinh, M., Korniienko, A., and Scorletti, G. (2013).
Embedding of uncertainty propagation : application to hierarchical performance analysis.
IFAC Symposium on System, Structure and Control, 5(1) :190–195.

Dinh, M., Korniienko, A., and Scorletti, G. (2014).
Convex hierarchicalrchical analysis for the performance of uncertain large scale systems.
IEEE Conference on Decision and Control, pages 5979– 5984.

Laib, K., Korniienko, A., Scorletti, G., and Morel, F. (2015).
Phase IQC for the hierarchical performance analysis of uncertain large scale systems.
IEEE Conference on Decision and Control (to appear).

Network Description of [Andersen et al., 2014]

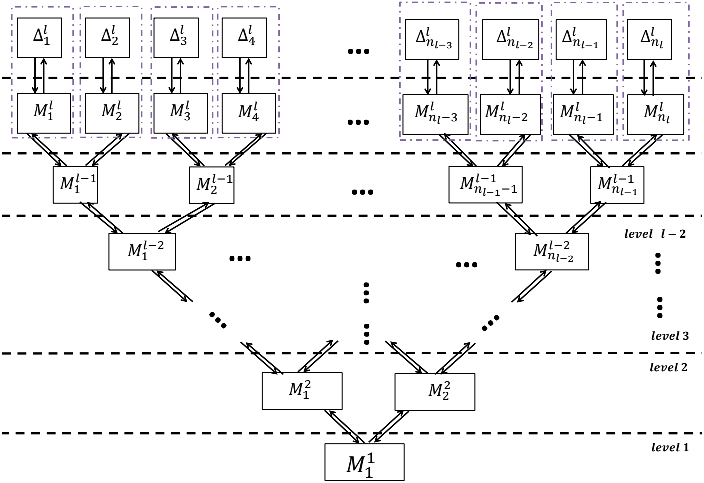
Network with N systems randomly generated



- Each system T_i is randomly generated with one parametric uncertainty
  - Nominally ($\Delta_i = 0$) stable
  - Robustly ($\Delta_i \neq 0$) stable
- For $i = 2, \ldots, N-1$
  - Each system T_i is MIMO (2 inputs/2 outputs)
  - Each system T_i is connected to T_{i-1} and to T_{i+1}
- T_1 and T_N are SISO
- The network
  - Nominally stable
  - Robustly stable

Network of [Andersen et al., 2014] : Used Hierarchical Approach

Multi level hierarchical approach



$\Longrightarrow$ Parallel computing at each level